SEQUENCE & SERIES

EXERCISE – I HINTS & SOLUTIONS

Sol.1 **D**

$a = P^2 + 1$, $d = 1$, $n = 2P + 1$

$$S_{2P+1} = \frac{2P+1}{2}[2(P^2+1) + (2P+1-1)\,1]$$

$$= \frac{(2P+1)}{2}[2P^2 + 2 + 2P] = (2P+1)(P^2+P+1)$$

$$= 2P^3 + 3P^2 + 3P + 1 \quad = P^3 + (P+1)^3$$

Sol.2 **D**

$a_1, a_2, a_3, \ldots\ldots$ in A.P.

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$6a_1 + (74 - 5)\,d = 225$

$$3(2a_1 + 23d) = 225 \Rightarrow (2a_1 + 23d) = \frac{225}{3}$$

$$a_1 + a_2 + \ldots. + a_{24} = \frac{24}{2}[2a_1 + 23d]$$

$$= 12 \times \frac{225}{3} = 900$$

Sol.3 **C**

Sum of divisible by 2

$$S_2 = \frac{50}{2}[2 + 100] = 25 \times 102 = 2550$$

Sum of divisible by 5

$$S_5 = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$$

Sum of divisible by 2 & 5

$$S_{2,5} = \frac{10}{2}[10 + 100] = 5 \times 110 = 550$$

required sum = 2550 + 1050 – 550 = 3050

Sol.4 **A**

$$S_k = \frac{k}{2}[2a + (k-1)\,d],\ S_x = \frac{x}{2}[2a + (x-1)d]$$

$$\&\ S_{kx} = \frac{kx}{2}[2a + (kx-1)d]$$

$$\frac{S_{kx}}{S_x} = \frac{k[2a+(kx-1)d]}{[2a+(x-1)d]} = \frac{k[(2a-d)+kxd]}{[(2a-d)+xd]}$$

= Independent of x if

$$2a - d = 0 \qquad \Rightarrow a = \frac{d}{2}$$

Sol.5 **D**

$$x \in R,\ (5^{1+x} + 5^{1-x}),\ \frac{a}{2},\ 25^x + 25^{-x}$$

$$2\,.\,\frac{a}{2} = (5^{1+x} + 5^{1-x}) + (25^x + (25^{-x}))$$

$\Rightarrow \quad a = 5\,(5^x + 5^{-x}) + (5^{2x} + 5^{-2x})$

$\because \quad 5^x + 5^{-x} \ge 2 \quad \& \quad 5^{2x} + 5^{-2x} \ge 2$

$\Rightarrow \quad a \ge 5.2 + 2 \qquad \Rightarrow \qquad a \ge 12$

Sol.6 **B**

$$3.\ A_1, A_2 = \ldots.\ , A_n, 54, \&\ \frac{A_8}{A_{n-2}} = \frac{3}{5}$$

$$d = \frac{54-3}{n+1} = \frac{51}{n+1}$$

$A_8 = 3 + 8d\ \&\ A_{n-2} = 3 + (n-2)\,d$

$$\frac{A_8}{A_{n-2}} = \frac{3+8d}{3+(n-2)d} = \frac{3}{5}$$

$$\Rightarrow \frac{3+8\left(\frac{51}{n+1}\right)}{3+(n-2)\left(\frac{51}{n+1}\right)} = \frac{3}{5}$$

$$\Rightarrow \frac{n+1+136}{n+1+(n-2)17} = \frac{3}{5}$$

$\Rightarrow 5(n + 137) = 3\,(18n - 33) \qquad \Rightarrow 784 = 49n$

$$\Rightarrow n = \frac{784}{49} = \frac{112}{7} \qquad \Rightarrow n = 16$$

Sol.7 **B**

$ar^2 = 4$

$a \cdot ar \cdot ar^2 \cdot ar^3 \cdot ar^4 = a^5 r^{10} = (ar^2)^5 = 4^5$

Sol.8 **B**

Sum of G.P. is possible $\Rightarrow |r| < 1$

$$S = \frac{a}{1-r} \Rightarrow r = \left(1 - \frac{a}{S}\right)$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a\left(1-\left(1-\frac{a}{S}\right)^n\right)}{\frac{a}{S}} = S\left[1-\left(1-\frac{a}{S}\right)^n\right]$$

Sol.9 D

$$\frac{1}{\log_2 4} + \frac{1}{\log_4 4} + \frac{1}{\log_8 4} + ... + \frac{1}{\log_{2^n} 4}$$

$= \log_4 2 + \log_4 2^2 + \log_4 2^3 + ... + \log_4 2^n$

$= (\log_4 2)\ [1 + 2 + 3 + ... + n]$

$$= \frac{1}{2} \times \frac{n(n+1)}{2} = \frac{n(n+1)}{4}$$

Sol.10 B

$$a_1 = 2,\ \&\ \frac{a_{n+1}}{a_n} = \frac{1}{3} = r$$

$$\sum_{r=1}^{20} a_r = \frac{a_1(1-r^{20})}{1-r} = \frac{2\left(1-\left(\frac{1}{3}\right)^{20}\right)}{\frac{2}{3}} = 3\left(1-\frac{1}{3^{20}}\right)$$

Sol.11 C

$x^2 - 3x + a = 0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}$ $\qquad x^2 - 12x + b = 0 \begin{smallmatrix}\gamma \\ \delta\end{smallmatrix}$

$\alpha, \beta, \gamma, \delta$ in increasing G.P.

Let $\alpha, \alpha r, \alpha r^2, \alpha r^3$

$\alpha + \alpha r = 3 \qquad \& \qquad \alpha r^2 + \alpha r^3 = 12$

$\alpha(1 + r) = 3 \qquad \alpha r^2 (1 + r) = 12$

$r^2 . 3 = 12$

$\therefore \quad \alpha = \frac{3}{1+r} \qquad r^2 = 4$

$\alpha = \frac{3}{3} \Rightarrow \alpha = 1\ r = 2, \quad (r = -2 \text{ reject})$

$\because$ G.P. increasing

$\alpha(\alpha r) = a \qquad \& \qquad (\alpha r^2)(\alpha r^3) = b$

$a = 1 \cdot 2 \qquad b = \alpha^2 r^5$

$a = 2 \qquad b = 1 \cdot 2^5 = 32$

Sol.12 A

$$3 + \frac{1}{4}(3 + d) + \frac{1}{4^2}(3 + 2d) + ... + \infty = 8$$

$a = 3,\ r = \frac{1}{4}$

(A.G.P.)

$$S_\infty = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$\Rightarrow 8 = \frac{3}{(3/4)} + \frac{d\left(\frac{1}{4}\right)}{3^2/4^2} \Rightarrow 8 = 4 + \frac{4d}{3^2}$$

$$\Rightarrow 4 = \frac{4d}{3^2} \quad \Rightarrow d = 3^2 \quad \Rightarrow d = 9$$

Sol.13 B

$$A = \frac{a+b+c}{3},\ G = (a\ b\ c)^{1/3},\ H = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$$

$$H = \frac{3(abc)}{ab+bc+ca}$$

$x^3 - (a + b + c)\ x^2 + (ab + bc + ca)\ x - abc = 0$

$\Rightarrow x^3 - 3Ax^2 + (3G^3/H)x - G^3 = 0$

Sol.14 C

$a^x = b^y = c^z = d^t = k$ (let)

$x \log a = \log k, \quad y \log b = \log k$

$z \log c = \log k, \quad t \log d = \log k$

a, b, c, d in G.P.

$\Rightarrow \log a, \log b, \log c, \log d$ in A.P.

$\Rightarrow \frac{\log k}{x}, \frac{\log k}{y}, \frac{\log k}{z}, \frac{\log k}{t}$ in A.P.

$\Rightarrow \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{t}$ in A.P. $\Rightarrow$ x, y, z, t in H.P.

Sol.15 B

$$\sum_{r=2}^{\infty} \frac{1}{r^2-1} = \sum_{r=2}^{\infty} \frac{1}{(r-1)(r+1)} = \frac{1}{2}\sum_{r=2}^{\infty}\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$$

$$= \frac{1}{2}\left[1-\frac{1}{3}+\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{5}+....+\frac{1}{n-1}-\frac{1}{n+1}+....\right]$$

$$n \to \infty \ \Rightarrow \ \frac{1}{n+1} \to 0$$

$$= \frac{1}{2}\left[1+\frac{1}{2}\right] = \frac{3}{4}$$

Sol.16 A

$x_i > 0,\ i = 1, 2, ..., 50$

$x_1 + x_2 + x_3 + ... + x_{50} = 50$

or $\sum_1^{50} x_i = 50 \Rightarrow \frac{\Sigma x_i}{50} = 1$

A.M. $\geq$ H.M.

$$\frac{\left(\sum_1^{50} x_i\right)}{50} \geq \frac{50}{\left(\sum_1^{50}\frac{1}{x_i}\right)} \Rightarrow 1 \geq \frac{50}{\left(\sum_1^{50}\frac{1}{x_i}\right)}$$

$$\Rightarrow \sum_1^{50}\frac{1}{x_i} \geq 50 \quad \text{Min value of } \Sigma\frac{1}{x_i} = 50$$

Sol.17 A

$a, a_1, a_2, a_3, \ldots.\ a_{2n}, b$ in A.P.

& $a, g_1, g_2, g_3, \ldots., g_{2n}, b$ in G.P.

$(a + b) = (a_1 + a_{2n}) = (a_2 + a_{2n-1}) = \ldots..$

& $ab = g_1g_{2n} = g_2g_{2n-1} = \ldots..$

$$\therefore\ \frac{a_1 + a_{2n}}{g_1g_{2n}} + \frac{a_2 + a_{2n-1}}{g_1g_{2n-1}} + \ldots. + \frac{a_n + a_{n+1}}{g_ng_{n+1}}$$

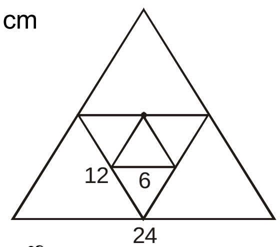
$$= \left(\frac{a+b}{ab}\right) + \left(\frac{a+b}{ab}\right) + \ldots. + \left(\frac{a+b}{ab}\right) \text{ (n times)}$$

$$\left\{h = \frac{2ab}{a+b}\right. \qquad \Rightarrow \frac{a+b}{ab} = \frac{2}{h}$$

$$= n\left(\frac{a+b}{ab}\right) = n \cdot \frac{2}{h} = \frac{2n}{h}$$

Sol.18 A

Perimeter $P_1 = 3 \times 24$ cm



$P_2 = 3 \times \frac{24}{2}$ cm

$P_3 = 3 \times \frac{24}{2^2}$ cm

$S_\infty = P_1 + P_2 + P_3 + \ldots.\ \infty$

$$= 3 \times 24\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots.\right]$$

$= 3 \times 24 \times 2 = 144$ cm

Sol.19 D

$a_k = a_{k+1} + a_{k+2} \qquad \forall\ a_k > 0$

$\Rightarrow ar^{k-1} = ar^k + ar^{k+1} \qquad \Rightarrow\ r > 0$

$\Rightarrow 1 = r + r^2 \qquad \Rightarrow r^2 + r - 1 = 0$

$$\Rightarrow r = \frac{-1 \pm \sqrt{1+4}}{2} \qquad \{r = -\text{ve reject}\}$$

$$\Rightarrow r = \frac{\sqrt{5}-1}{2} = 2\left(\frac{\sqrt{5}-1}{4}\right) = 2 \sin 18^\circ$$

Sol.20 C

$$S = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.\ \infty = \frac{\pi^2}{6}$$

$$\text{Now } S_{even} = \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots.\ \infty$$

$$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.\infty\right] = \frac{1}{2^2}\ \frac{\pi^2}{6} = \frac{\pi^2}{24}$$

$$S_{odd} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots.\ \infty$$

$$= S - S_{even}$$

$$= \frac{\pi^2}{6} - \frac{\pi^2}{24} = \frac{3\pi^2}{24} = \frac{\pi^2}{8}$$

Sol.21 C

$d = (a_2 - a_1) = (a_3 - a_2) = \ldots. = (a_n - a_{n-1} \neq 0)$

sind [cosec a_1 cosec a_2 + coseca_2 cosec a_3 + ... + cosec a_{n-1} cosec a_n]

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} + \ldots + \frac{\sin d}{\sin a_{n-1} \sin a_n}$$

$$= \sum_{r=2}^{n} \frac{\sin(a_r - a_{r-1})}{\sin a_{r-1} \sin a_r}$$

$$= \sum_{r=2}^{n} \left(\frac{\sin a_r \cos a_{r-1}}{\sin a_{r-1} \sin a_r} - \frac{\cos a_r \sin a_{r-1}}{\sin a_{r-1} \sin a_r}\right)$$

$$= \sum_{r=2}^{n} (\cot a_{r-1} - \cot a_r) = \cot a_1 - \cot a_n$$

Sol.22 A

$S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots.\ 2002^2 + 2003^2$

$= 1 + (3^2 - 2^2) + (5^2 - 4^2) + \ldots. + (2003^2 - 2002^2)$

$= 1 + 2 + 3 + 4 + 5 + \ldots. + 2002 + 2003$

$$= \frac{2003}{2}\,[1 + 2003] = 2003\,(1002)$$

$= (2000 + 3)\,(1000 + 2) = 2007006$

Sol.23 A

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots. + \frac{1}{n}$$

$$1 + \frac{3}{2} + \frac{5}{3} + \ldots. + \left(\frac{2n-1}{n}\right)$$

$$= \sum_{n=1}^{n}\left(\frac{2n-1}{n}\right) = \sum_{n=1}^{n}\left(2 - \frac{1}{n}\right) = \sum_{n=1}^{n} 2 - \sum_{n=1}^{n}\frac{1}{n}$$

$$= 2n - \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}\right) = 2n - H_n$$

Sol.24 C

$$S_1 = \sum_1^n n \,,\; S_2 = \sum_1^n n^2 \,,\; S_3 = \sum_1^n n^3$$

$$S_1 = \frac{n(n+1)}{2} \,,\; S_2 = \frac{n(n+1)(2n+1)}{6} \,,\; S_3 = \frac{n^2(n+1)^2}{4}$$

$$\therefore \quad \frac{S_3(1+8S_1)}{S_2^2} = \frac{n^2(n+1)^2(1+4(n+1)n)}{\left(\dfrac{4n^2(n+1)^2(2n+1)^2}{36}\right)}$$

$$= \frac{9(1+4n^2+4n)}{(2n+1)^2} = \frac{9(2n+1)^2}{(2n+1)^2} = 9$$

Sol.25 C

$a_1, a_2, \ldots, a_n$ in H.P.

$\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$ in A.P.

$$\therefore \quad d = \frac{a_1 - a_2}{a_1a_2} = \frac{a_2 - a_3}{a_2a_3} = \ldots \frac{a_{n-1} - a_n}{a_{n-1}a_n}$$

$a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n$

$$= \frac{d}{d}\,[a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n]$$

$$= \frac{1}{d}\,[(a_1 - a_2) + (a_2 - a_3) + \ldots + (a_{n-1} - a_n)]$$

$$= \frac{a_1 - a_n}{d}$$

$$\left\{\left(\frac{1}{a_n}\right) = \left(\frac{1}{a_1}\right) + (n-1)d\right.$$

$$\Rightarrow \frac{a_1 - a_n}{a_1a_n} = (n-1)d \Rightarrow \frac{(a_1 - a_n)}{d} = (n-1)\,a_1a_n$$

Sol.26 A

$$x = \sum_{n=0}^{\infty} a^n \,,\; y \sum_{n=0}^{\infty} b^n \,,\; z = \sum_{n=0}^{\infty} c^n \quad \begin{cases} |a| < 1 \\ |b| < 1 \\ |c| < 1 \end{cases}$$

$$x = \frac{1}{1-a} \,,\; y = \frac{1}{1-b} \,,\; z = \frac{1}{1-c}$$

a, b, c in A.P.

$1-a, 1-b, 1-c$ in A.P.

$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ in H.P. $\Rightarrow$ x, y, z in H.P.

Sol.27 D

$$x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$$

$$(x)^2 + (3y)^2 + (5z)^2 - 3xy - 15yz - 5zx = 0$$

$$\Rightarrow \frac{1}{2}\,[(x-3y)^2 + (3y-5z)^2 + (5z-x)^2] = 0$$

$\Rightarrow x = 3y \;\&\; 3y = 5z \;\&\; 5z = x$

$\Rightarrow x = 3y = 5z \;\Rightarrow\; y = \frac{x}{3},\; z = \frac{x}{5}$

$x, y, z \Rightarrow x, \frac{x}{3}, \frac{x}{5}$

We know 1, 3, 5 in A.P. $x \neq 0$

$\Rightarrow \frac{1}{x}, \frac{3}{x}, \frac{5}{x}$ in A.P. $\Rightarrow x, \frac{x}{3}, \frac{x}{5}$, in H.P.

$\Rightarrow$ x, y, z in H.P.

Sol.28 A

$1(1!) + 2(2!) + 3(3!) + \ldots + n(n!)$

$$\sum_{r=1}^{n} r(r!) = \sum_1^n (r+1-1)r! = \sum_1^n [(r+1)r! - r!]$$

$$= \sum_1^n [(r+1)! - r!] = (n+1)! - 1! = (n+1)! - 1$$

Sol.29 A

S = 1 . (2 + 3 + 4 + ... + n) +
+ 2 (3 + 4 + 5 + ... + n) +
+ 3 (4 + 5 + ... + n) +
- - - - - - - - - - - - - - - - - -
+ (n – 2) [(n + 1) + n]
+ (n – 1) (n)

$$S = \frac{(1+2+3+4+\ldots+n)^2 - (1^2+2^2+3^2+\ldots+n^2)}{2}$$

$$= \frac{1}{2}\left[\frac{n^2(n+1)^2}{2^2} - \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \frac{1}{24}\,n(n+1)\,[3n^2 + 3n - 4n - 2]$$

$$= \frac{1}{24}\,n(n+1)(3n^2 - n - 2) = \frac{1}{24}\,n(n+1)(n-1)(3n+2)$$

Aliter

1 . 2 + 1.3 + 1.4 + ... + 1. n
+ 2.3 + 2.4 + ... + 2. n
+ 3.4 + ... + 3. n

S = (1 . 2) + (1 + 2) 3 + (1 + 2 + 3) .4 + ...
+ [1 + 2 + 3 +.......+ (n – 1)]n

$T_n = [1 + 2 + 3 + \ldots (n-1)]\,n$

$$= \Sigma\,\frac{n^2(n-1)}{2} = \frac{1}{2}\,(\Sigma n^3 - \Sigma n^2)$$

Sol.30 A

$\sqrt{2} + \sqrt{6} + \sqrt{18} + \sqrt{54} + ... +$ (10 terms)

$= \sqrt{2}\,(1 + \sqrt{3} + \sqrt{9} + \sqrt{27} + ... +$ (10 terms))

$= \sqrt{2}\,(1 + 3^{1/2} + 3^{1} + 3^{3/2} + ... +)$

$$= \sqrt{2}\ \ 1.\ \frac{(1-(\sqrt{3})^{10})}{(1-\sqrt{3})} = \frac{\sqrt{2}((\sqrt{3})^{10}-1)}{(\sqrt{3}-1)}$$

$$= \frac{\sqrt{2}(3^5-1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{2}}{2}\ 242\,(\sqrt{3}+1)$$

$= \sqrt{2}\,(121)\,(\sqrt{3}+1) = 121\,(\sqrt{6}+\sqrt{2})$

Sol.31 A

$$\frac{1}{(1+P)} - \frac{1-P}{(1+P)^2} + \frac{(1-P)^2}{(1+P)^3}\ .....$$

$$-1 < r = -\left(\frac{1-P}{1+P}\right) < 1 \qquad \because\ P > 0$$

$$= \frac{a}{1-r} = \frac{\left(\frac{1}{1+P}\right)}{1+\frac{1-P}{1+P}} = \frac{1}{1+P+1-P} = \frac{1}{2}$$

Sol.32 C

a, G_1, G_2, b & a, A, b

$b = ar^3 \Rightarrow r = \left(\frac{b}{a}\right)^{1/3} \qquad \because\ 2A = a + b$

$G_1 = ar = a\left(\frac{b}{a}\right)^{1/3} \qquad \because\ G_1G_2 = ab$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{2/3}$

$$\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = \frac{G_1^3+G_2^3}{G_1\,G_2} = \frac{a^3\frac{b}{a}+a^3\frac{b^2}{a^2}}{ab}$$

$$= \frac{a^2b+ab^2}{ab} = a + b = 2A$$

Sol.33 C

$a_n = x^{1/2^n} + y^{1/2^n}$ & $b_n = x^{1/2^n} - y^{1/2^n}$, $\forall\ n \in N$

$\Rightarrow \quad a_nb_n = (x^{1/2^n})^2 - (y^{1/2^n})^2$

$\Rightarrow \quad a_nb_n = x^{1/2^{n-1}} - y^{1/2^{n-1}} = b_{n-1}$

$$a_1a_2a_3 \ldots a_n \times \frac{b_1b_2b_3\ldots b_n}{b_1.b_2.b_3\ldots b_n}$$

$$= \frac{(a_1b_1)(a_2b_2)(a_3b_3)\ldots(a_nb_n)}{b_1.b_2.b_3\ldots b_n}$$

$$= \frac{(x-y)(b_1)(b_2)(b_3)\ldots(b_{n-1})}{b_1b_2b_3\ldots b_{n-1}.b_n} = \frac{x-y}{b_n}$$

Sol.34 D

$S = 2.2^2 + 3.2^3 + 4.2^4 + ... + n.2^n$

$2S = \qquad 2.2^3 + 3.2^4 + ... + (n-1)2^n + n.2^{n+1}$

$\Rightarrow\ -S = 2^3 + 2^3 + 2^4 + ... + 2^n - 2.n^{n+1}$

$\Rightarrow -S = 1 + (1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^4) - n.2^{n+1}$

$$-S = 1 + \frac{1(2^{n+1}-1)}{2-1} - n.2^{n+1}$$

$-S = 1 + 2^{n+1} - 1 - n.2^{n+1}$

$S = n.2^{n+1} - 2^{n+1} = 2^{n+1}(n-1) = 2^n\,.\,2^{10}$

$\Rightarrow \quad n - 1 = 2^9 \Rightarrow n = 1 + 512 \Rightarrow n = 513$

Sol.35 A

$1^2 + 2^2 + 3^2 + ... + 2003^2 = (2003)\,(4007)\,(334)$

& $1\,(2003) + 2\,(2002) + 3\,(2001) + ... + 2003\,(1)$

$= (2003)\,(334)\,(x)$

$$\Rightarrow \sum_{r=1}^{2003} r[2003-(r-1)] = (2003)\,(334)\,x$$

$$\Rightarrow \sum r[2004-r] = (2003)\,(334)\,x$$

$$\Rightarrow 2004\sum_{1}^{2003} r - \sum_{1}^{2003} r^2 = (2003)\,(334)\,x$$

$$\Rightarrow \frac{(2003)(2004)^2}{2} - (2003)\,(4007)(334)$$

$= (2003)\,(334)\,x$

$\Rightarrow (2004)\,(1002) - (4007)\,(334) = (334)x$

$\Rightarrow 6(1002) - (4007) = x$

$\Rightarrow x = 6012 - 4007 = 2005$

Sol.36 C

If $x > 0$

$\log_2 x + \log_2(\sqrt{x}) + \log_2\sqrt[4]{x} + \log_2\sqrt[8]{x} + \log_2\sqrt[16]{x} + ... = 4$

$\Rightarrow \log_2 x + \log_2 x^{1/2} + \log_2 x^{1/4} + ... = 4$

$$\Rightarrow (\log_2 x)\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right] = 4$$

$$\Rightarrow (\log_2 x)\ \frac{1}{1-\frac{1}{2}} = 4 \Rightarrow \log_2 x = 2 \Rightarrow x = 4$$

Sol.37 C

$$\sum_{r=1}^{n} t_r = \frac{1}{12} n(n+1)(n+2) = S_n$$

$t_r = S_n - S_{n-1}$

$$= \frac{n(n+1)(n+2)}{12} - \frac{(n-1)n(n+1)}{12}$$

$$= \frac{n(n+1)}{12}[n+2-n+1]$$

$$t_r = \frac{n(n+1)}{4} \Rightarrow \frac{1}{t_r} = \frac{4}{n(n+1)}$$

$$\sum_{r=1}^{n} \frac{1}{t_r} = 4\sum_{r=1}^{n}\frac{1}{n(n+1)} = 4\sum\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= 4\left[1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots + \frac{1}{n} - \frac{1}{n+1}\right] = \frac{4n}{n+1}$$

Sol.38 A

a, b, c in A.P. $\Rightarrow$ $2b = a + c$

p, q, r, in H.P. $\Rightarrow$ $q = \frac{2pr}{p+r}$

ap, bq, cr in G.P. $\Rightarrow$ $b^2q^2 = acpr$

$$\Rightarrow \frac{b^2 . 4(pr)^2}{(p+r)^2} = ac\,pr \Rightarrow \frac{(a+c)^2 pr}{(p+r)^2} = ac$$

$$\Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac} \Rightarrow \frac{p^2+r^2}{pr} + 2 = \frac{a^2+c^2}{ac} + 2$$

$$\Rightarrow \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}$$

Sol.39 C

common diff. = d, in A.P.

$T_7 = 9 \Rightarrow a + 6d = 9 \Rightarrow a = (9 - 6d)$

$T_1T_2T_7 = a\,.\,(a + d)\,9 = (9 - 6d)(9 - 5d)\,9$

$= 9\,(30d^2 - 99d + 81) = 27\,(10d^2 - 33d + 27)$

Min value at $d = \frac{-(-33)}{2 \cdot 10} = \frac{33}{20}$

Sol.40 C

$$H = \frac{16}{5} = \frac{2ab}{a+b},\ A = \frac{a+b}{2},\ G = \sqrt{ab}$$

$$\frac{16}{5} = \frac{2G^2}{2A} \Rightarrow 5G^2 = 16A$$

& Given $2A + G^2 = 26$

$\therefore\ 5(26 - 2A) = 16A$

$\Rightarrow 5 \cdot 26 = 26A \Rightarrow A = 5$

$\therefore\ a + b = 10$

$$\frac{16}{5} = \frac{2ab}{10} \Rightarrow ab = 16 \Rightarrow a = 2,\ b = 8$$

Sol.41 B

$1^2 + 2^2 + \ldots n^2 = 1015$

$$\frac{n(n+1)(2n+1)}{6} = 1015$$

(A) $n = 15 \Rightarrow \frac{15 \times 16 \times 31}{6} = 1240 \Rightarrow n \neq 15$

(B) $n = 14 \Rightarrow \frac{14 \times 15 \times 29}{6} = 1015 \Rightarrow n = 14$

Sol.42 A

$S_1 = \frac{n}{2}[2 + (n-1)1]$

$S_2 = \frac{n}{2}[4 + (n-1)3]$

$S_3 = \frac{n}{2}[6 + (n-1)5]$

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$S_p = \frac{n}{2}[2p + (n-1)(2p-1)]$

$S_1 + S_2 + S_3 + \ldots + S_p = \frac{n}{2}[(2 + 4 + 6 + \ldots + 2p)]$

$+ (n-1)(1 + 3 + 5 + \ldots + (2p-1)]$

$= \frac{n}{2}[p(p+1) + (n-1)p^2]$

$= \frac{n}{2}[p^2 + p + np^2 - p^2] = \frac{np(1+np)}{2}$

Sol.43 B

$T_p = a,\ T_q = b$

$a = A + (p-1)d$

$b = A + (q-1)d \Rightarrow$ subtract $\frac{a-b}{p-q} = d$

add $a + b = 2A + (p + q - 1)d - d$

$\Rightarrow 2A + (p + q - 1)d = (a + b) + d$

$$S_{p+q} = \frac{(p+q)}{2}[2A + (p+q-1)d]$$

$$= \frac{(p+q)}{2}\left[a + b + \frac{a-b}{p-q}\right]$$

Sol.44 D

$S = 1 + 2 + 3 + \ldots + 100 = 5050$

by 3, $S_3 = \frac{33}{2}[3+99] = \frac{33 \times 102}{2} = 33 \times 51 = 1683$

by 5, $S_5 = \frac{20}{2}[5 + 100] = 10 \times 105 = 1050$

by 3 & 5, $S_{15} = \frac{6}{2}[15 + 90] = 3 \times 105 = 315$

by 3 or 5, $S_{3or5} = S_3 + S_5 - S_{15}$

$= 1683 + 1050 - 315 = 2418$

Not divisible by 3 or 5 = 5050 – 2418 = 2632